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Photon–Photon Angular Correlation in e^+e^- Collision in QED

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The angular correlation of two monochromatic photons produced in e^+e^- collision (annihilation), defined as the average of the cosine of the angle of separation θ between their outgoing momenta, is studied completely in momentum space to lowest order in the fine structure constant in QED. This is done from the expression of the e^+e^--momentum-spin, $\gamma\gamma$-polarization-averaged conditional probability density for the angular distributions of the photons, given that the process has occurred. Although the density is in general nonvanishing for θ less than as well as greater than $\pi/2$, the angular correlation is found to be strictly negative for all energies, indicating that the photons tend to travel in opposing directions in a statistical sense. The back-to-back motion, however, is very quickly unfavored as we move to higher energies.

1. INTRODUCTION

Much attention has been given (Dicke, 1954; Ernst and Stehle, 1968; Mostowski and Sobolewska, 1983; Mandel, 1983; Richter, 1983; Stendel and Richter, 1987; Duncan and Stehle, 1987; Bialynicki-Birula and Bialynicka-Birula, 1990; Manoukian, 1992; Manoukian and Ungkitchanukit, 1994) to the tendency of photons produced by various sources to form collimated beams, a result which is generally attributed to the Bose character of the photons making them travel in the same directions. A statistical term for the study of such an angular correlation was introduced (Bialynicki-Birula and Bialynicka-Birula, 1990) defined as the average of the cosine of the angle between the directions of the momenta of two outgoing photons. A positive angular correlation would indicate that the photons have, in a statistical sense, the tendency to travel in the same

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direction, and a negative one that they travel in opposing directions. In the present paper we investigate the fate of two monochromatic photons produced in e^+e^- collision (annihilation) to lowest order in the fine structure constant in QED. This gives a nontrivial mechanism of photons produced by interacting parent "sources." This is perhaps the simplest process that we can think of that can be calculated in a realistic quantum field theory setting, at least to lowest order, which also due to the constraint imposed on the momenta of the photons produced by the "sources" complicates quite a bit the general argument of a positive angular correlation for the photons due to their Bose character. Unlike our earlier analysis (Manoukian and Ungkitchanukit, 1994), we study the process completely in momentum space with no restrictions on the angular distributions of the outgoing two photons. The angular correlation is computed from the expression of the e^+e^- momentum-spin-averaged, $\gamma\gamma$-polarization-averaged, conditional probability density *given* that the process has occurred. We find that at all energies, the angular correlation is strictly negative, indicating that the photons tend, in a statistical sense, to travel in opposing directions. The analysis also indicates that the back-to-back motion becomes very quickly unfavored as we move to higher energies. We hope that this result of negative correlation can be tested experimentally.

2. ANGULAR CORRELATION OF THE PHOTONS

To lowest order in the fine structure constant α, the amplitude for the process $e^+e^- \to \gamma\gamma$, is, up to unimportant factors for the problem at hand, in a standard notation

$$\alpha^2 \bar{v}(p_2, \sigma_2)\left\{e_\mu(k_1, \lambda_1)\gamma^\mu \frac{1}{\gamma(p_1 - k_2) + m} \gamma^\nu e_\nu(k_2, \lambda_2) + (k_1, \lambda_1) \leftrightarrow (k_2, \lambda_2)\right\} u(p_1, \sigma_1) \tag{1}$$

Here we consider the process with $k_1^0 = k_2^0 \equiv k^0$. The e^+e^--spin (σ_1, σ_2)-averaged, $\gamma\gamma$-polarization (λ_1, λ_2)-averaged probability density is readily obtained to be (Sokolov *et al.*, 1988)

$$\alpha^2\left[\frac{m^4(k_1k_2)^2}{2(p_1k_1)^2(p_1k_2)^2} + \frac{m^2(k_1k_2)}{(p_1k_1)(p_1k_2)} + 1 - \frac{(k_1k_2)^2}{2(p_1k_1)(p_1k_2)}\right] \tag{2}$$

We consider all processes with different incoming momenta for the e^+e^- pair and perform the momentum averages over all such processes which conserve the total four-momentum. That is, we multiply (2) by the invariant measure

$$\frac{d^3\mathbf{p}_1}{p_1^0}\frac{d^3\mathbf{p}_2}{p_2^0}\,\delta(p_1+p_2-k_1-k_2)$$

and integrate to obtain in a standard manner (Sokolov *et al.*, 1988) for the probability density in equation

$$f_{Q^2}(\theta)=N(Q^2)\left(\left[1+\frac{1}{x}-\frac{1}{2x^2}\right]\ln[2(x^2-x)^{1/2}+2x-1]\right.$$
$$\left.-(x^2-x)^{1/2}\left[\frac{1}{x}+\frac{1}{x^2}\right]\right) \tag{3}$$

where

$$x=\frac{Q^2}{2}(1-\cos\theta)\geq 1 \tag{4}$$

$$\mathbf{k}_1\cdot\mathbf{k}_2=(k^0)^2\cos\theta, \qquad Q^2=(k^0)^2/m^2 \tag{5}$$

and $N(Q^2)$ is a normalization factor such that

$$\int_{-1}^{1} f_{Q^2}(\theta)\Theta(x-1)\,d(\cos\theta)=1 \tag{6}$$

where $\Theta(x-1)$ is the Heaviside step function. The constraint $x\geq 1$ arises as a consequence of momentum conservation $k_1+k_2=p_1+p_2$.

The probability density $f_{Q^2}(\theta)$ is plotted in Fig. 1 for various values of Q^2, as a function of θ in degrees. It is in general nonvanishing for θ less than as well as greater than $\pi/2$.

The angular correlation of the two photons is defined by

$$\langle c(Q^2)\rangle=\int_{-1}^{1}\cos\theta\, f_{Q^2}(\theta)\Theta(x-1)\,d(\cos\theta) \tag{7}$$

The angular correlation versus Q^2 is plotted in Figs. 2 and 3, and has been studied numerically for arbitrarily large Q^2. The angular correlation $\langle c(Q^2)\rangle$ is strictly negative, indicating that the photons have, in a statistical sense, the tendency to travel in opposing directions. As also seen from these graphs, the back-to-back motion becomes very quickly unfavored as we move up in energy. We hope that this result of negative correlation can be tested experimentally. At present it is not clear to what extent the other interactions should play a role in our analysis, which was carried fully within the context of QED. What about higher-order contributions to $\langle c(Q^2)\rangle$ within QED? Higher-order contributions to $\langle c(Q^2)\rangle$ will simply modify the expression for the latter to an expression of the form $c_1+O(\alpha)$,

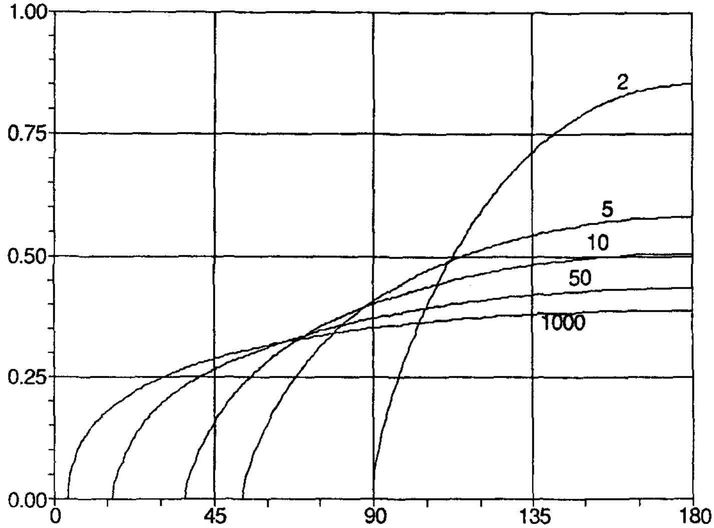

Fig. 1. Plot of the probability density $f_{Q^2}(\theta)$ versus θ in degrees, for $Q^2 = 2, 5, 10, 100, 1000$.

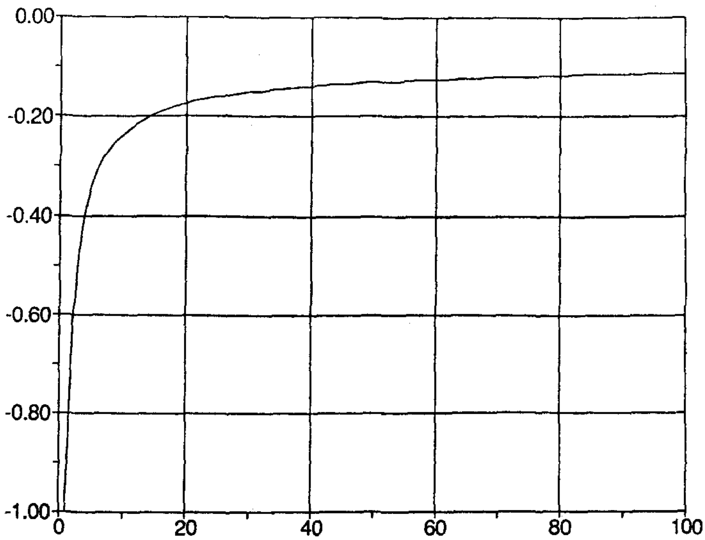

Fig. 2. Plot of the angular correlation $\langle c(Q^2)\rangle$ versus Q^2 for $Q^2 \leq 100$.

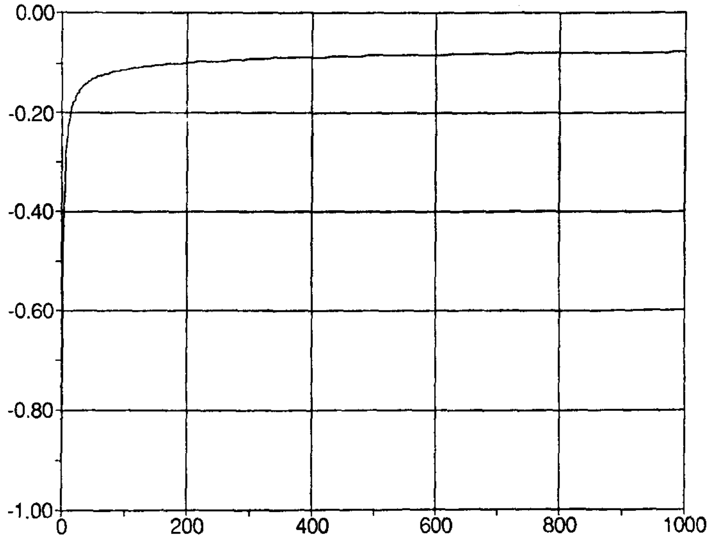

Fig. 3. Plot of the angular correlation $\langle c(Q^2)\rangle$ versus Q^2 for $Q^2 \leq 1000$.

where c_1 is computed on the right-hand side of (7). Since c_1 is found to be not too small in magnitude, it is expected that at least for energies not too high such higher-order contributions will not modify our conclusions.

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